DOCUMENT RESUME

ED 187 584 SE 031 103

TITLE	The Arithmetic Project Course for Teachers - 3. Topics: Parentheses and "Multiplying Before You Add," Standstill Points. Supplement: Answers to Common Questions About the Film "Standstill Points."
INSTITUTION	Education Development Center, Inc., Newton, Mass.: Illinois Univ., Urbana.
SPONS AGENCY	Carnegie Corp. of New York, N.Y.; National Science Foundation, Washington, D.C.
PUB DATE	73
NOTE	19p.; For related documents, see SE 031 100-120. Contains light and broken type.
EDRS PRICE	MF01/PC01 Plus Postage.
DESCRIPTORS	*Algorithms; Elementary Education; *Elementary School Mathematics; *Elementary School Teachers; Films; Grade 5; Inservice Education; *Inservice Teacher Education; Mathematics Curriculum; *Mathematics Instruction; Mathematics Teachers; Number Concepts; *Problem Sets; Teacher Education
IDENTIFIERS	*University of Illinois Arithmetic Project

ABSTRACT

This is one of a series of 20 booklets designed for participants in an in-service course for teachers of elementary mathematics. The course, developed by the University of Illinois Arithmetic Project, is designed to be conducted by local school personnel. In addition to these booklets, a course package includes films showing mathematics being taught to classes of children, extensive discussion notes, and detailed guides for correcting written lessons. This booklet contains: exercises on using parentheses, multiplying before adding, and standstill points; a summary of the problems from the film "Standstill Points," and the supplement. (MK)

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THE ARITHMETIC PROJECT COURSE FOR TEACHERS

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TOPICS: Parentheses and "Multiplying Before You Add". Standstill Points.

FILM: Standstill Points, Grade 5

SUPPLEMENT: Answers to Common Questions About the Film "Standstill Points"

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NAME: .....

SE 031 103

This booklet is part of a course for teachers produced by The Arithmetic Project in association with Education Development Center. Principal financial support has come from the Carnegie Corporation of New York, the University of Illinois, and the National Science Foundation.

The course is available from:

THE ARITHMETIC PROJECT
Education Development Center
55 Chapel Street
Newton, Massachusetts 02160

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BOOK THREE

I. SOME NOTES ABOUT PARENTHESES, AND ABOUT WHAT TO DO WHEN THEY AREN'T THERE

What is $3 + 5 \times 6$? Is it 33 or 48 ?

If you have $10 + 3 + 1$, it doesn't matter whether you do $10 + 3$ first, or $3 + 1$:

$$(10 + 3) + 1 = 14$$

$$10 + (3 + 1) = 14$$

Now do these:

1. $100 + (1 \times 2) =$

2. $(100 + 1) \times 2 =$

3. $(3 \times 3) + 3 =$

4. $3 \times (3 + 3) =$

5. $(5 \times 5) \times 5 =$

6. $5 \times (5 \times 5) =$

Notice how the loop clarifies such an expression as $(10 - 3) \times 4$.

You do the $10 - 3$ part all by itself. The result is what's multiplied by 4 . This use of a loop will help you and your students understand parentheses.

$(10 - 3) \times 4$ is the same as $(10 - 3) \times 4$ and $(10 - 3 \times 4) + 1$ is the same as $((10 - 3) \times 4) + 1$.

Possibly parentheses originated from such loops:

$$(10 - 3) \times 4 = (10 - 3) \times 4 = (10 - 3) \times 4 = (10 - 3) \times 4$$

More problems:

7. $20 - (5 \div 3) =$

8. $(20 - 5) \div 3 =$

The question now arises, what to do if you find something without parentheses. The meaning of

$$3 + 30 + 7\frac{1}{2}$$

is clear: $(3 + 30) + 7\frac{1}{2}$ is the same as $3 + (30 + 7\frac{1}{2})$. But what about this:

$$3 + 4 \times 10$$

Does it mean $(3 + 4) \times 10$ or $3 + (4 \times 10)$ or is it meaningless as it stands, just as $3 \times = \div 55$ is meaningless?

The answer is that, by agreement among people who use mathematics,

$$3 + 4 \times 10$$

is understood to mean the same as

$$3 + (4 \times 10)$$

or,

$$3 + (4 \times 10)$$

A loop or parentheses may be considered to be around the numbers to be multiplied. Similarly, parentheses are implicit around numbers to be divided:

$$20 - 5 \div 3 \quad = \quad 20 - (5 \div 3)$$

It is accordingly incorrect merely to do things in the order given. One must consider separately the numbers to be multiplied or divided.

A way to remember this convention is to think of multiplication as a shorthand way of writing an addition problem. Thus,

$$3 + 4 \times 10$$

may be considered

$$3 + 10 + 10 + 10 + 10$$

Once written this way, the grouping no longer matters, so parentheses are unnecessary. In the case of division, $30 - 51 \div 17$ may be thought of as $30 - \frac{51}{17}$, which is unambiguous.

Sometimes this convention is stated as "multiply or divide before you add or subtract."

9. $10 + 4 + 6 + 3 \times 17 =$ _____

10. $(4 + 3) \times 3 + 2 \times 5 =$ _____

11. $4 + 3 \times 3 + 2 \times 5 =$ _____

12. $100 - \frac{1}{2} =$ _____

13. $100 - 1 \div 2 =$ _____

14. $(100 - 1) \div 2 =$ _____

15. $1 \div 2 - 100 =$ _____

16. $(6 \times 2) \div 4 =$ _____

17. $6 \times (2 \div 4) =$ _____

18. $6 \times 2 \div 4 =$ _____

19. $3 \times (\frac{1}{2} + \frac{1}{2}) \times 3 =$ _____

20. $3 + (\frac{1}{2} + \frac{1}{2}) - 3 =$ _____

21. $(3 + \frac{1}{2}) - (3 + \frac{1}{2}) =$ _____

22. $3 + \frac{1}{2} - 3 - \frac{1}{2} =$ _____

23. $3\frac{1}{2} - 3\frac{1}{2} =$ _____

24. $3 + \frac{1}{2} - 3 + \frac{1}{2} =$ _____ (The answer is not 0.)

25. $3 + \frac{1}{2} + \frac{1}{2} - 3 =$ _____

26. $3 - 3 + \frac{1}{2} - \frac{1}{2} =$ _____

27. $\frac{1}{2} + 3 - 3 - \frac{1}{3} =$ _____

28. $\frac{1}{3} + 3 - 3 - \frac{1}{2} =$ _____

29. $\frac{1}{3} + 3 - (3 + \frac{1}{2}) =$ _____

If you make it a point to write with this spacing:

$$3 \cdot + \ 2 \times 9$$

both you and your students will understand it better. Problem 11, for example, is better written thus:

$$4 + 3 \times 3 + 2 \times 5 = 23$$

Finally, one might inquire about the meaning of a construction like

$$10 \div 2 \times 5$$

This could be interpreted as

$$(10 \div 2) \times 5 = 25$$

or

$$10 \div (2 \times 5) = 1$$

Whichever of these alternative meanings is intended ought to be specified. It might be preferable to write

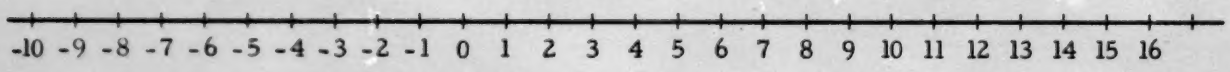
$$\frac{10}{2} \times 5 = 25 \qquad \text{or} \qquad \frac{10}{2 \times 5} = 1$$

As a student continues further in mathematics, the use of the division bar will predominate.

II. STANDSTILL POINTS

1. $\square \longrightarrow 3 \times \square - 8$

Where applicable, draw all jumps for the following questions on the number line. Don't forget the arrowhead showing the direction of each jump. (If you put some care into drawing jumps neatly, for the time being, you and your students will more easily see some of the characteristics of different types of jumping rules. Later on, neatness will be less important, although drawing jumps on a number line will remain a valuable way of looking at rules.)



(a) Start at 5 . One jump. Land? _____

(b) Start at 6 . One jump. Land? _____

(c) Start at 2 . One jump. Land? -2

(d) Start at 3 . One jump. Land? _____

(e) Start at 1 . One jump. Land? _____

(f) A common wrong answer to (e) above is -4 . What question might someone who makes this mistake have been answering?

(g) Where can you start so that you land at 0 ? _____

(h) Consider the lengths of the jumps you have drawn. How many jumps are two units long? _____ How many jumps are four units long? _____ Find another place to start to make another jump six units long (different from the jump starting at 1).
Start: _____ Land: _____

Continue with the rule $\square \longrightarrow 3 \times \square - 8$:

(i) Start at 10 . Make one jump. Land? _____

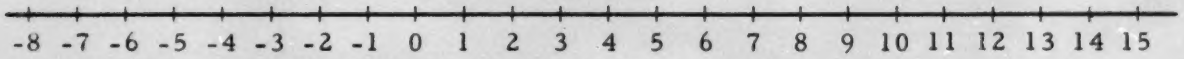
Spaces moved? _____

(j) Using this rule, from what starting point can you make another jump of the same length as in (i) but going in the opposite direction ? _____ Landing point: -14

(k) Find a starting place for a jump 0 spaces long (where the landing point is the same as the starting point.) _____ (We shall refer to this point as the "standstill point" for this rule.)

2. Rule: $\square \longrightarrow 3 \times \square - 4$

Draw the jumps for (a) through (e) on the line below.



(a) Start at 5 . One jump. Land? _____

(b) Start at -1 . One jump. Land? _____

(c) Start at 0 . One jump. Land? _____

(d) Start at 4 . One jump. Land? _____

(e) Start at _____ One jump. Land? -1

(f) Draw whatever other jumps you need to find the standstill point.

Standstill point: _____ (If you start at $1\frac{1}{3}$, you land on 0. $1\frac{1}{3}$ is not the standstill point.)

3. Find the standstill point for this rule. Draw jumps if you wish.

$\square \longrightarrow 3 \times \square - 7$

Standstill point: _____

☆4. (a) What other method besides looking at the pattern of jumps might a child suggest as a quick way to find standstill points for the rules in the foregoing problems?

(b) See if this method works to find the standstill point of

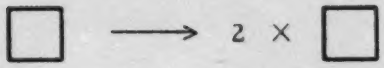
$\square \longrightarrow 3 \times \square + 120$

Standstill point: _____

If this method doesn't work, how can you fix it so that it will work for both rules?

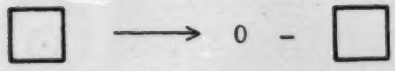
☆Throughout this institute, problems marked with a star (☆) are wholly optional. They are included for those who may want an additional challenge.

☆5. Find the standstill point or points of each of the following rules:

(a)  Standstill point(s): _____

(b) Standstill point(s): _____

(c) Standstill point(s): _____

(d)  Standstill point(s): _____

Epilogue

You may have been taught that when an expression requires several symbols of aggregation, one uses different symbols. For example, you may be accustomed to seeing

$$4 \times \left\{ [\ 7 \times (3 + 2) + 5\ ] + 10 \right\}$$

using brackets and braces, etc. There is no difference mathematically between what is written above and what is written here

$$4 \times ((\ 7 \times (3 + 2) + 5\ ) + 10\ )$$

and this is the form you will usually see during the course. In your work on scratch paper you may find it helpful to make "outside" parentheses larger, such as

$$4 \times \Big(\big(\ 7 \times (3 + 2) + 5\ \big) + 10\ \Big)$$

but in printing it is much easier to use parentheses of the same size.

* * *

In discussing how to simplify expressions such as $2 \times 4 + 3$, one often hears the statement "multiply before you add". If used correctly, this advice will lead to correct answers. But, strictly, the notion of time and what you do <u>first</u> can be misleading. The convention that applies here is <u>group the multiplication correctly</u>. Thus $2 \times 4 + 3$ does equal $(2 \times 4) + 3$ and not $2 \times (4 + 3)$. Time and what you thought of first have nothing to do with it.

* * *

The term "standstill point" was invented by students working with this topic. Mathematicians use the term "fixed point" instead. Predicting whether a transformation has a fixed point and if so, where, is an important topic in mathematics at many levels. The fixed point(s) of a given transformation tell a lot about the roots of some corresponding equation -but we are not going to pursue this idea now.

Summary of Problems in the Film "Standstill Points"

5th Grade, James Russell Lowel School, Watertown, Massachusetts
Teacher: David A. Page

Here is a jumping rule: $\square \longrightarrow 3 \times \square$

Start at 4; make one jump. Where do you land? (12)

Make a jump from 2. Where do you land? From 0? From $\frac{1}{2}$?

How long is the jump from 4 to 12? From 2 to 6? From $\frac{1}{2}$ to $1\frac{1}{2}$? From 0?

Take a jump from z2. Land?

Now a jump from $z3\frac{1}{2}$. Land?

Using $\square \longrightarrow 3 \times \square - 5$:

Who can tell where to start so that you land on 1? (2)

How long is that jump?

Who can tell where to start to land on 10?

Find another place to start besides 2 where there is a jump that is 1 unit long. (0 is suggested; this jump lands at z5 and that is the companion of the jump from 5 to 10) (Answer: 3)

Try starting at z2. Land? (z11)

How long is the jump from z2?

Find a jump where you get back to the same place. $(2\frac{1}{2})$

(Wrong answers: $1\frac{1}{2}$, 4)

New Rule: $\square \longrightarrow 3 \times \square - 19$

Where is the place that you start from to land right back where you started? $(9\frac{1}{2})$ (Wrong answers computed: 100, 38, $6\frac{1}{3}$, 9, $9\frac{1}{3}$, $1\frac{1}{2}$.)

$\square \longrightarrow 3 \times \square - 117$

According to your method, what would be the place that you start from in order to get right back there? $(58\frac{1}{2})$

$\square \longrightarrow 4 \times \square - 18$

Make a jump from 5. Where do you land?

Make a jump from 10. Landing point?

Where can we start and land right back there? (Wrong answer: 18)

Using $\square \longrightarrow 4 \times \square - 24$, where can we start to land back at the same place?

With $\square \longrightarrow 4 \times \square - 31$, what is the starting point that gets us right back where we started? $(10\frac{1}{3})$

Now I will change the end number: $\square \longrightarrow 4 \times \square - z15$

What is the standstill point?

(z5 was suggested.)

Here are some problems to help you do z20 - z15:

10 - 5 = 5
10 - 4 = 6
10 - 3 = 7
10 - 2 = 8
10 - 1 = 9
10 - 0 = 10
10 - z1 = 11
10 - z2 = 12
10 - z5 = 15

Now we'll change the first number: $\square \longrightarrow 10 \times \square - 18$

What is the point that stands still for this rule?

New Rule: $\square \longrightarrow 7 \times \square - 22$

What is the starting point that takes you right back where you started? $(3\frac{2}{3})$

(Wrong answer: $3\frac{1}{3}$)

Can anyone tell me what his method is? (The children stated that they understood ways of finding standstill points from having done examples of this type: $\square + \square + \square = \square + 8$.)

Supplement

Answers to Questions About the Film
"Standstill Points"

Q: How did the teacher introduce negative numbers, and why were they labelled "z" numbers?

A: This class was introduced to negative numbers on an earlier day by working with the rule $\square \longrightarrow \square - 5$ and making successive jumps. Soon the rule required moving to the other side of zero.

When a jump had been made into the region of the number line below zero, this class was asked what it would like to call the numbers there. Nancie said "zero-one, zero-two, zero-three," which she suggested might be written 01, 02, 03. The teacher took the word "zero" and used the abbreviations z1, z2, z3, thereby avoiding the confusion with decimals that might have occurred with Nancie's symbolism. He worked with this terminology for two class hours. The third day he told the children that henceforth he would call the numbers below zero negative numbers and write -1, -2, -3, and so on.

Other classes have selected the letter b (for "below") as a symbol: b1, b2, b3, or 1b, 2b, 3b, and so forth.

Many children have heard about negative numbers, if only from other children. Notice that one of the virtues of working with these unconventional symbols is that they can help make clear the distinction between a "-" used for the operation of subtraction, and a "-" used to mean "the opposite of". A child who has done z10 - z5 is in a better position to see what is going on when he first faces -10 - (-5).

Q: By the end of the class not more than a dozen children seemed to know what was going on. Do the others ever learn a method for finding standstill points for a variety of jumping rules?

A: Normally the teacher would work much more than a single hour with this topic, and on successive days other children would begin to see what was happening. During the next hour the class might do more of the kind of work they were doing at the end of this hour, with the teacher calling on children other than Brian, Bruce, Helen and Christine. Also, a written work sheet might be appropriate.

In time the teacher and students might want to try to find standstill points for more complicated rules, including some with more than one standstill point and some with no standstill points at all.

Q: The film said that the instructor worked on number lines with this class two previous times. Can you give a synopsis of what happened then?

A: In the first session with number lines, the class made jumps with such rules as $\square \longrightarrow \square + 3$, $\square \longrightarrow \square + 5$, and $\square \longrightarrow \square + 1\frac{1}{2}$. The rule $\square \longrightarrow \square - 5$ was used to jump into the negatives. The following day the children investigated such questions as where are all the numbers on which $\square \longrightarrow \square + 5$ and $\square \longrightarrow \square + 3$ both land, starting at 0 for example.

Q: Shouldn't the teacher seek generalizations as soon as possible in order to get on with the subject and to give practice in verbalization?

A: It is true that the teacher may need to encourage some children to describe their ways of solving problems, because the previous experience of many children may have discouraged them from admitting to anything but the "correct" way of doing something. The discovery that there may be more than one good way to solve a problem comes too often as a surprise.

However, if generalizations are sought too quickly, slower children are deprived of the chance to hunt for fast methods of their own. The teacher may want to get to general methods sooner in some cases and later in others. As with much in education, a mixed strategy is probably best.

If the slower children have had a chance to get answers to a series of problems, even if only by mechanical procedures, the probability is increased that these children will appreciate ingenious ways of solving the problems. This, of course, is the first step toward becoming interested in finding clever ideas.

Finally, another reason to avoid asking for generalizations prematurely is that the teacher may thereby cut off the search for other methods.

Q: At one point in the film, when the rule is $\square \longrightarrow 3 \times \square - 5$, the teacher asks for the other jump with a length of one space. How is a classroom teacher supposed to know things like the fact that $\square \longrightarrow 3 \times \square - 5$ has exactly two jumps of one space?

A: As more and more of the jumps are drawn, the symmetric pattern becomes clearer: any two points the same distance from the standstill point are starting points for jumps of equal length, one jump to the right and the other to the left. But the teacher need not have this knowledge before presenting jumping rules in the classroom.